

1 June 03 Model Solutions

①  $R_A = R_C = R$

a)  $R + R = 80g + 40g \Rightarrow 2R = 120g \Rightarrow R = 60gN$

b)  $M(A): \begin{cases} R \times 3 = 80g \times x + 40g \times 2 \\ 180g = 80g + 80g \times x \\ \frac{100}{80} = x \Rightarrow x = 1.25m \end{cases}$

②  $I = mv - mu$

a)  $I = mv - mu = 0.12 \times 3 - 0.12 \times 0 = 0.36 \text{ Ns}$

b)  $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$   
 $0.12 \times 3 + 0.08 \times 0 = 0.12 \times 1.2 + 0.08 v$   
 $0.36 = 0.144 + 0.08 v$   
 $\Rightarrow v = 2.7 \text{ ms}^{-1}$

c) Take  $\rightarrow$  to be positive.  
 $I = mv - mu = 1.2 \times 0.12 - 3 \times 0.12 = -0.216 \text{ Ns}$  Impulse is  $0.216 \text{ Ns}$

③  $u = -4$   
 $v = v$   
 $a = 9.8$   
 $s = +5$   
 $t = ? \Rightarrow v = 10.7 \text{ ms}^{-1}$

b)  $s = ut + \frac{1}{2}at^2 \Rightarrow 5 = -4t + \frac{1}{2} \times 9.8t^2$   
 $\Rightarrow 4.9t^2 - 4t - 5 = 0$   
 $\Rightarrow t = \frac{4 \pm \sqrt{16 + 4 \times 4.9 \times 5}}{9.8} = \frac{4 + \sqrt{144.2}}{9.8} = 1.50 \text{ s}$

c) We have ignored air resistance, the size of the diver, the horizontal component of velocity, spinning in the air, hitting the board on the way down.

③  $\mu = 0.4$   
 $F_r = 0.4R$   
 $R = 3g \cos 30^\circ \therefore F_r = 1.2g \cos 30^\circ = 10.2 \text{ N}$

b)  $u = 6$   
 $v = 0$   
 $a = a$   
 $s = 3$   
 $t = ?$

To find  $a$ :  $RF = ma$   
 $-Fr - 3g \sin 30^\circ = 3a$   
 $\Rightarrow \frac{-1.2g \cos 30^\circ - 3g \sin 30^\circ}{3} = a$   
 $\Rightarrow a = -8.29 \text{ ms}^{-2}$

$v^2 = u^2 + 2as \Rightarrow 0 = 36 + 2 \times -8.29 \times 3$   
 $\Rightarrow 16.6s = 36$   
 $\Rightarrow s = 2.17 \text{ m}$

④  $\text{Area under A} = \frac{1}{2} \times 12 \times 60 = 360$   
 $\text{Area under B} = \frac{1}{2} \times 40 \times 60 = 1200$

b) At  $T$ , both travelled same distance,  $\therefore$  area under graphs is equal.  
 $(A) = \frac{1}{2} \times 30 \times (T + (T-12)) = 15(T+T-12) = 30T - 180$   
 $(B) = \frac{1}{2} \times 60 \times ((T-40) + (T-64)) = 30(2T-104) = 60T - 3120$

Since  $(A) = (B)$ ,  $30T - 180 = 60T - 3120$   
 $2940 = 30T$   
 $\Rightarrow T = 98 \text{ s}$

④  $\alpha = \tan^{-1}(\frac{1}{4}) \Rightarrow \cos \alpha = \frac{4}{5}$   
 $\sin \alpha = \frac{3}{5}$   
 $F_r = \mu R$

$R = 5g \cos \alpha + 20 \sin \alpha = 4g + 12$   
 $F_r = \mu R = \mu(4g + 12)$

$F_r + 20 \cos \alpha = 5g \sin \alpha$   
 $\mu(4g + 12) + 20 \cos \alpha = 5g \sin \alpha$   
 $\mu = \frac{5g \sin \alpha - 20 \cos \alpha}{4g + 12} = \frac{3g - 16}{4g + 12} = \frac{67}{256} \approx 0.262$

⑤ a)  $\vec{r} = 2\hat{i} - 3\hat{j}$   
 $\vec{u} = -2\hat{i} + 7\hat{j}$   
 $\vec{v} = \vec{u} + \vec{a}t$   
 $\vec{a} = 2\hat{i} - 3\hat{j}$   
 $\vec{v} = (-2 + 2t)\hat{i} + (7 - 3t)\hat{j}$   
 $\frac{1}{2}t = t$

Parallel to  $\vec{u}$  means  $\vec{v}$  component = 0:  
 $7 - 3t = 0 \Rightarrow t = \frac{7}{3} \text{ s}$

b)  $t = 3$ ,  $\vec{v} = (-2 + 2 \times 3)\hat{i} + (7 - 3 \times 3)\hat{j} = 4\hat{i} - 2\hat{j}$   
 $\text{Speed} = |\vec{v}| = \sqrt{4^2 + 2^2} = \sqrt{20} \approx 4.47 \text{ ms}^{-1}$

c)  $t = 3$ :  $\alpha = \tan^{-1}(\frac{1}{4})$   
 $\text{Angle} = 90^\circ + \alpha = 116.6^\circ$

④  $400 \text{ N}$   
 $1200 \text{ kg}$   
 $2400 \text{ N}$

a)  $RF = ma$  Truck:  $2400 - 600 - T = 1200a$   
 $1800 - T = 1200a$   
 $\Rightarrow T = 1800 - 1200a$  (1)  
 Car:  $T - 400 = 800a$  (2)  
 Sub (1) into (2):  $(1800 - 1200a) - 400 = 800a$   
 $1400 = 2000a$   
 $a = 0.7 \text{ ms}^{-2}$

b)  $T = 1800 - 1200a$   
 $= 1800 - 1200 \times 0.7 = 960 \text{ N}$

c)  $600 \text{ N}$   
 $1200 \text{ kg}$   
 $2400 \text{ N}$

New acceleration:  $RF = ma$   
 $2400 - 600 = 1200a$   
 $1800 = 1200a$   
 $\Rightarrow a = 1.5 \text{ ms}^{-2}$

To reach  $28 \text{ ms}^{-1}$ :  $u = 20$   
 $v = 28$   
 $a = 1.5$   
 $s = ?$   
 $t = t$

$v = u + at \Rightarrow t = \frac{v-u}{a} = \frac{28-20}{1.5} = 5.33 \text{ s}$

If rope had not broken, then time taken to reach  $28 \text{ ms}^{-1}$ :  
 $u = 20$   
 $v = 28$   
 $a = 0.7$   
 $s = ?$   
 $t = t$

$v = u + at \Rightarrow t = \frac{v-u}{a} = \frac{28-20}{0.7} = 11.43 \text{ s}$

Difference between times =  $11.43 - 5.33 = 6.1 \approx 6 \text{ s}$  QED